

Unified study of structures and reactions In light neutron-rich systems

—Present results of ^{12}Be and future studies—

Makoto Ito

RIKEN Nishina Center, Theoretical Nuclear Physics Lab.

I. Introduction:

Some details of cluster model

Cluster model and (ab-initio) shell model

II. : Present study

Subject and Significance

Unified studies of structure in ^{12}Be and the $\alpha+^8\text{He}$ scattering
(Monopole Transition)

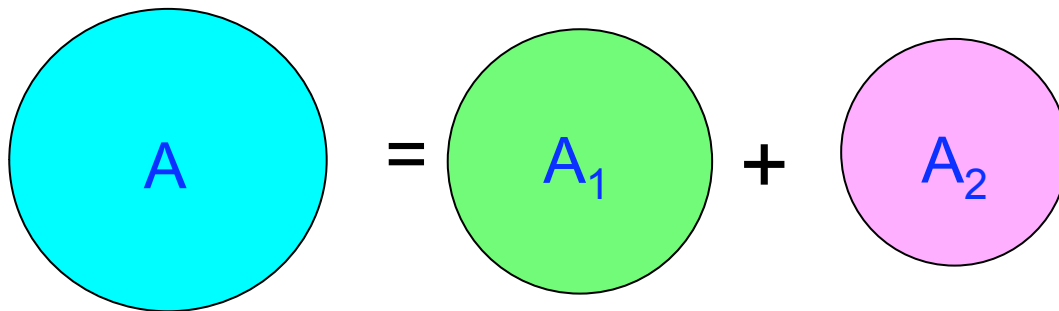
III. Summaries and future studies

Saturation Properties in Nuclei

1. Density $\rho \sim 0.18 \text{ fm}^{-3}$
2. Binding energy per nucleon $E/A \sim 8 \text{ MeV}$

⇒ Analogy to Liquid Drop (Easily sticking and separating)

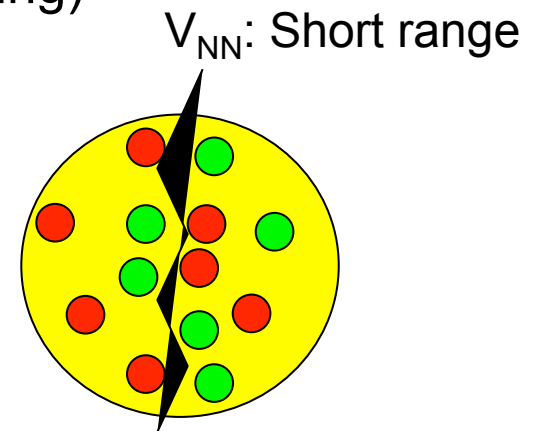
If the saturation property is strict,



$$E=8 \times A = E=8 \times (A_1 + A_2)$$

$\Delta E=0$

Separation without energy difference !



In realistic cases, ΔE is finite, but it is quite small value.
(α -separation, $\Delta E/A \ll 1 \text{ MeV}$)

Cluster structures in 4N nuclei

IKEDA Diagram

α separation: $\Delta E/A \ll E/A$

α -Particle \Rightarrow Stable

${}^3\text{H}+p \sim 20 \text{ MeV}$



Systematic Appearance of α cluster structures

Ikeda's Threshold rules

Molecular structures will appear close to the respective cluster threshold.



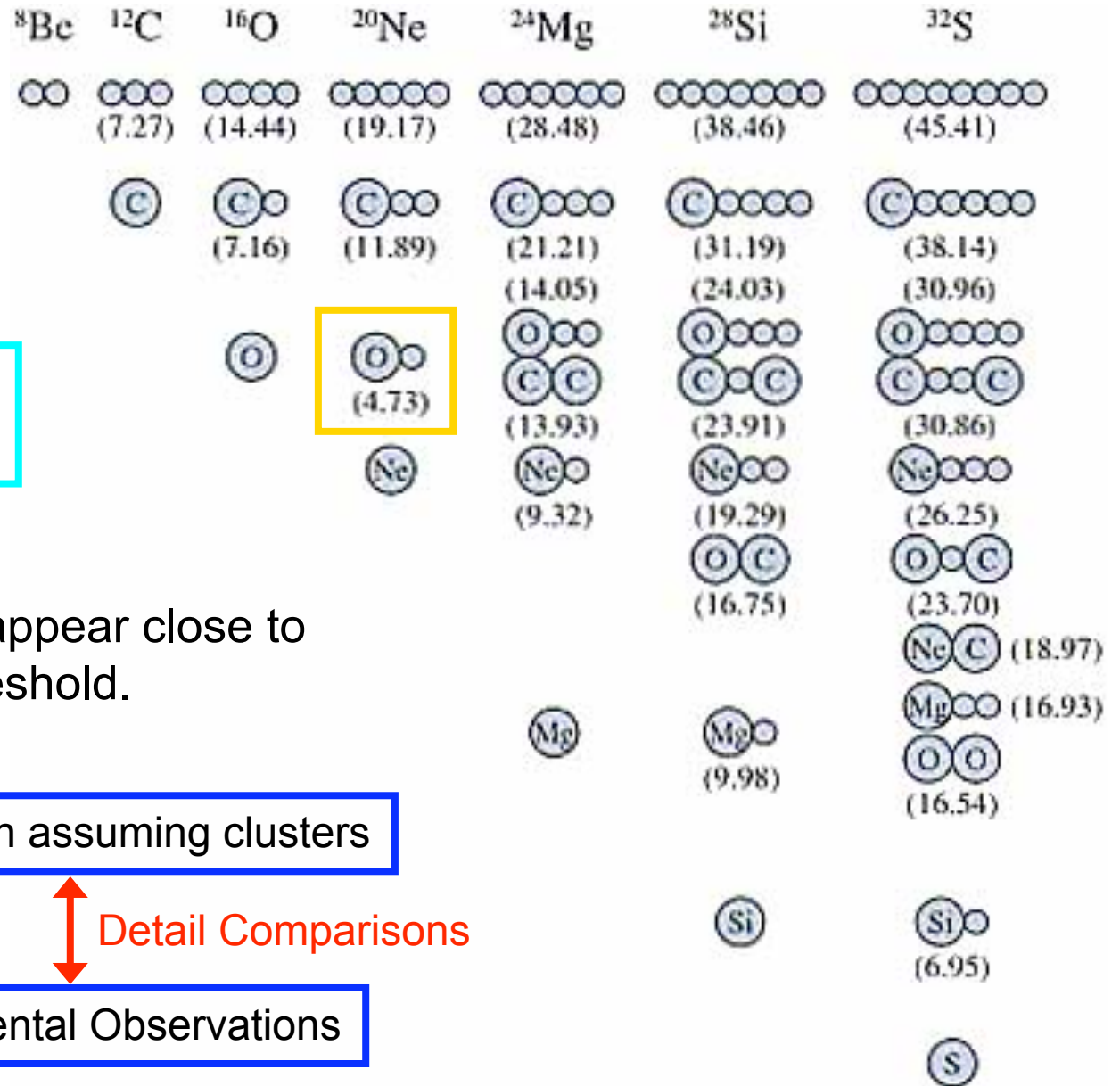
Established by

Calculation assuming clusters



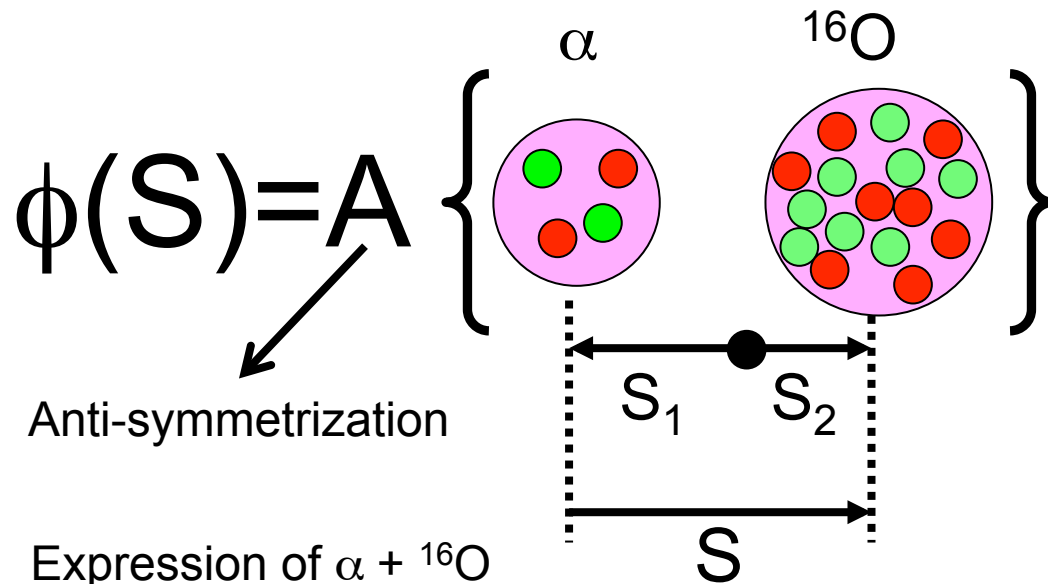
Detail Comparisons

Experimental Observations



Microscopic Cluster Model (GCM)

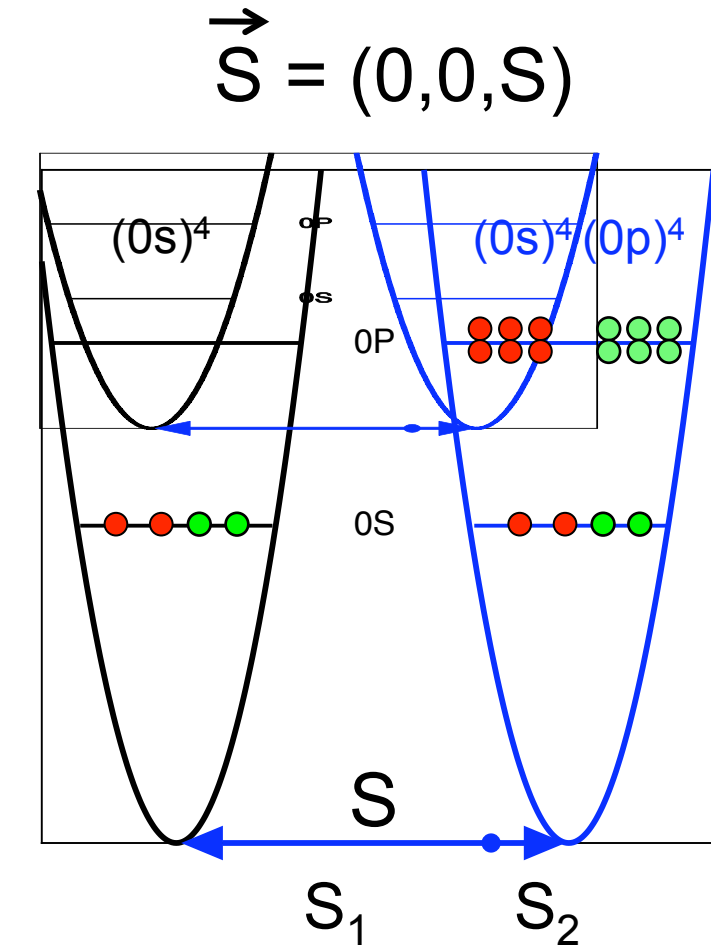
Sketch of $\alpha + {}^{16}\text{O}$



$$\phi(\mathbf{S}) = A \left\{ \varphi_{\alpha}(\xi, \mathbf{S}_1) \varphi_{16\text{O}}(\zeta, \mathbf{S}_2) \right\}_{\mathbf{S}}$$

Internal state of cluster: Direct products of harmonic oscillator w.f.

$$\varphi(\xi, \mathbf{S}) = \prod_i \chi(i) \quad \chi(i): \text{one particle w.f.} \sim H_{n_x, n_y, n_z}(\vec{r}_i - \vec{S}) \cdot \exp \left\{ -\nu (\vec{r}_i - \vec{S})^2 \right\}$$



Shifted harmonic oscillator

Separation of cluster wave function

$$\begin{aligned} \phi(\mathbf{S}) &= A \left\{ \varphi_\alpha(\xi, \mathbf{S}_1) \varphi_{160}(\zeta, \mathbf{S}_2) \right\}_{\mathbf{S}} \\ &= A \left\{ \underset{\text{C. M.}}{X_{\text{cm}}} \cdot \underset{\text{Relative}}{Y_{\text{rel}}(\vec{\mathbf{R}} - \vec{\mathbf{S}})} \cdot \underset{\text{Internal}}{\varphi_\alpha(\xi_{\text{in}}) \varphi_{160}(\zeta_{\text{in}})} \right\} \\ Y_{\text{rel}}(\vec{\mathbf{R}} - \vec{\mathbf{S}}) &\sim \exp \left\{ -\mu(\vec{\mathbf{R}} - \vec{\mathbf{S}})^2 \right\} \end{aligned}$$

Schrodinger (Hill-Wheeler) Equation

$$(H - E) \Psi = 0 \quad \Psi = \int d\vec{\mathbf{S}} f(\vec{\mathbf{S}}) \phi(\vec{\mathbf{S}})$$

$$H = \sum_{i=1}^A t_i - T_{c.m.} + \sum_{i < j}^A v_{ij}$$

In actual calculations, angular part of $\vec{\mathbf{S}}$ is integrated with an appropriate weight.

(Angular momentum projection)

Shell Model Limit of Cluster W.F.

Harmonic Oscillator Quanta

(x and y directions are omitted.)

(n_x, n_y, n_z)

$$\lim_{S_z \rightarrow 0} A \left\{ e^{-\nu z_1^2} e^{-\nu(z_2 - S_z)^2} \right\} (0,0,0) \otimes (0,0,0) \quad (p \uparrow, p \uparrow)$$

$$= \lim_{S_z \rightarrow 0} \left\{ e^{-\nu z_1^2} e^{-\nu(z_2 - S_z)^2} - e^{-\nu z_2^2} e^{-\nu(z_1 - S_z)^2} \right\}$$

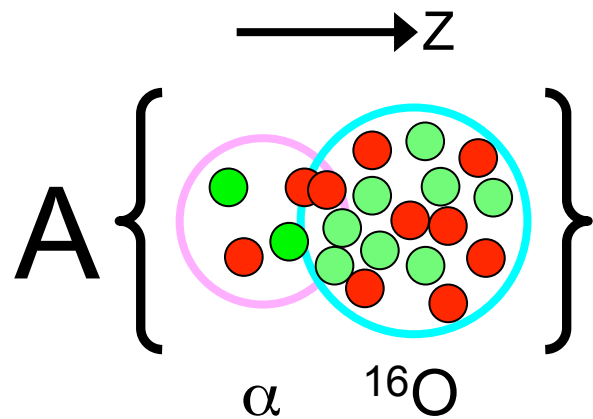
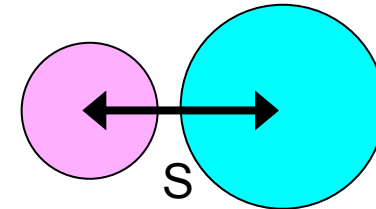
$$\approx e^{-\nu z_1^2} e^{-\nu z_2^2} (1 + 2\nu z_2 S_z) - e^{-\nu z_2^2} e^{-\nu z_1^2} (1 + 2\nu z_1 S_z)$$

$$= 2\nu S_z \left(e^{-\nu z_1^2} z_2 e^{-\nu z_2^2} - e^{-\nu z_2^2} z_1 e^{-\nu z_1^2} \right)$$

$$= 2\nu S_z A \left\{ e^{-\nu z_1^2} z_2 e^{-\nu z_2^2} \right\} (0,0,0)(0,0,1) \quad \text{Z-quantum number is increased.}$$

Shell model limit of $^{20}\text{Ne} = \alpha + ^{16}\text{O}$

We consider the $S=0$ limit of the $\alpha + ^{16}\text{O}$ system



Harmonic Oscillator Quanta: (n_x, n_y, n_z)

$(0,0,0)^4 \otimes (0,0,0)^4 (1,0,0)^4 (0,1,0)^4 (0,0,1)^4$

Increase of Z-quanta

$(0,0,0)^4 (1,0,0)^4 (0,1,0)^4 (0,0,1)^4 (0,0,2)^4$

Simple harmonic oscillator shell model of ^{20}Ne

($N=8$ for α -particle)

Cluster Limit in Shell model wave function

$u_n(x)$: Harmonic Oscillator in 1 dim. $u_0(x) = \left(\frac{2\nu}{\pi}\right)^{1/4} e^{-\nu x^2}$

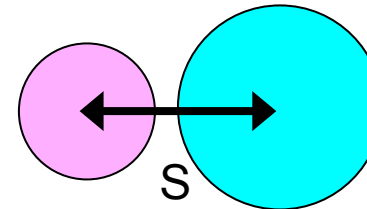
$$u_0(x - S) = \sum_{n=0}^{\infty} C(n) u_n(x)$$

$|C(n)|^2 = \left(\nu S^2\right)^n \frac{1}{n!} e^{-\nu S^2}$ Poisson Distribution $\langle n \rangle = \nu S^2$ $\Delta n = \sqrt{\nu S^2}$

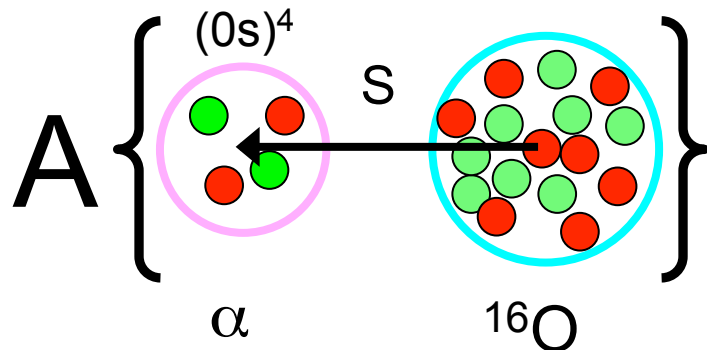
$S \Rightarrow \infty$ (Development of cluster) \Rightarrow Mixing of higher quanta

Cluster limit of $^{20}\text{Ne} = \alpha + ^{16}\text{O}$

Limit of $S \rightarrow$ Large in the $\alpha + ^{16}\text{O}$ system



Strong configuration mixing measured from a center of ^{16}O



$S \sim 5.1 \text{ fm}$ (Contact distance)
 $v \sim 0.24 \text{ fm}^{-2}$ (Standard value)

Quanta for single nucleon: n

$$\langle n \rangle = v S^2 \approx 6 \quad \Delta n = \sqrt{\langle n \rangle} \approx 2.4$$

Quanta for 4 nucleons (α -particle): N

$$\left. \begin{aligned} \langle N \rangle &= 4 \langle n \rangle \approx 24 \\ \Delta N &= \sqrt{4 \langle n \rangle} \approx 5 \end{aligned} \right\} \rightarrow$$

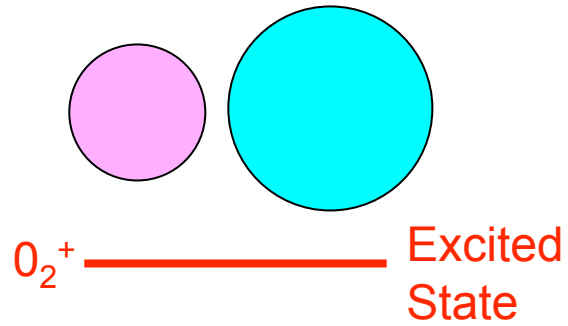
If $S \sim 7 \text{ fm}$, $N \sim 50!$

α particle is loosely coupled!

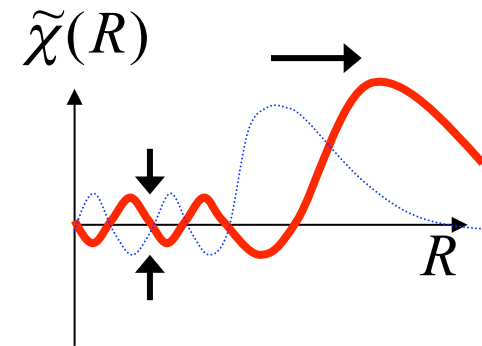
c.f. : No core shell model ($N < 10$) for $A < 16$

Behaviours of solutions: Example of ^{20}Ne

$$\Psi = A \left\{ \chi(\vec{R}) \varphi_\alpha(\vec{\xi}_1) \varphi_{16O}(\vec{\xi}_2) \right\}$$

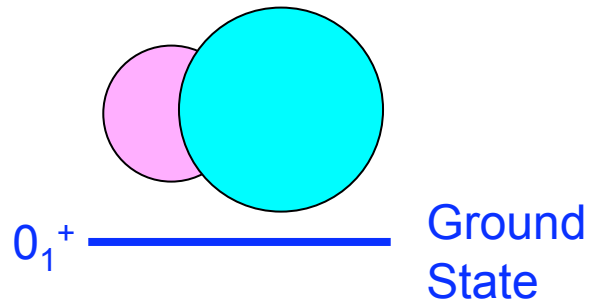


$$\Psi(0_2^+)$$

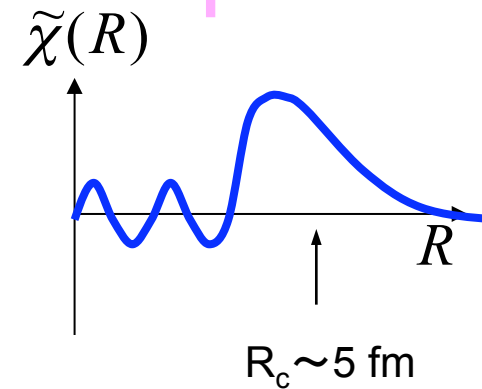


$$\langle \Psi(0_2^+) | \Psi(0_1^+) \rangle = 0$$

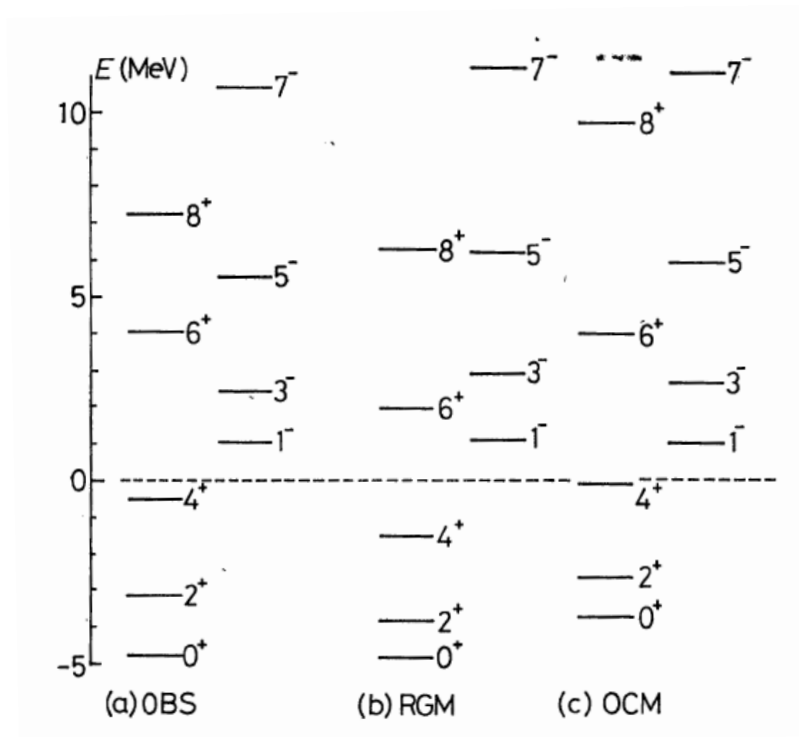
Cluster Excitation Mode



$$\Psi(0_1^+)$$



Application of Cluster Model to ^{20}Ne



RGM calculation of $\alpha + ^{16}\text{O}$

T. Matsuse, M. Kamimura,
Y. Fukushima, PTP53 (1975)

Inversion doublet in $\alpha + ^{16}\text{O}$

$$\Psi(\pm) = \left\{ \begin{array}{c} \text{Pink} \text{ } \text{Red} \\ \text{Red} \text{ } \text{Pink} \end{array} \right\} \pm$$

Scattering problem is also solvable.

Cluster formation from the ab-initio cal. with the realistic NN force

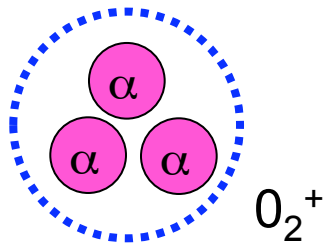
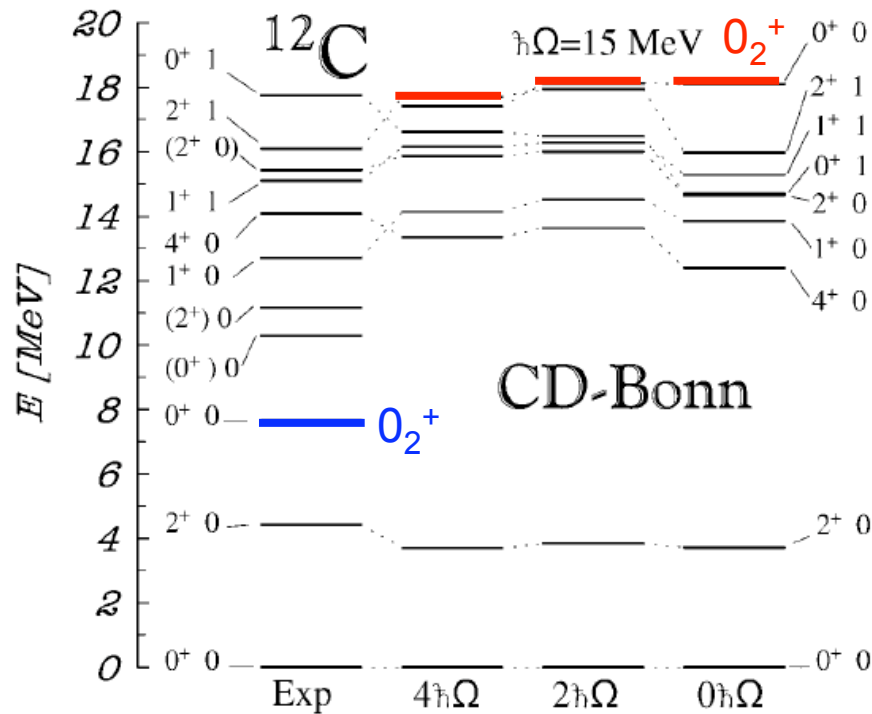
R.B.Wiringa et al., PRC62 (2000), Green Function Monte Carlo calculation ($A < 12$)

AV_{18} +Urbana 3body NN force $\Rightarrow {}^8\text{Be} = \alpha + \alpha$

Other systems : Only yrast states are reproduced (Cluster states cannot be handled).

第一原理計算の最近の進展 (II) : No core shell model計算

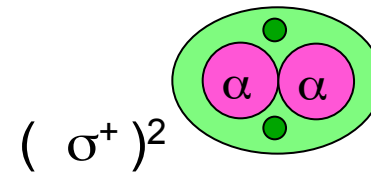
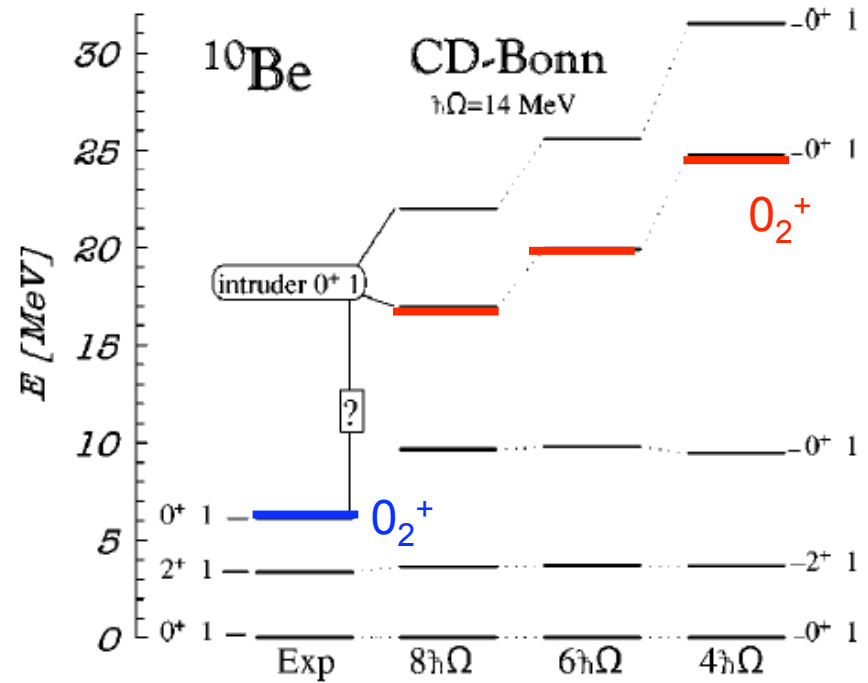
Navratil et al., PRL84 (00)



$N \sim 60$ for α - α rel. motion

By T. Yamada et al.

Caurier et al., PRC66 (02)



Singnificance of the present work

A. Present situation of the ab-initio calculations with realistic NN forces

1. ab-initio calculations can mainly reproduce the **yrast states in (well known) light systems.**
2. Application to the reaction problem is performed (e.g. ${}^4\text{He}+n$, $A < 6$).

K. M. Nolett et al., PRL99 (07)

However, these calculations are difficult to describe intruder states, in which **the cluster degree of freedoms strongly activate.**

(Predictive powers for unknown state is weak...)

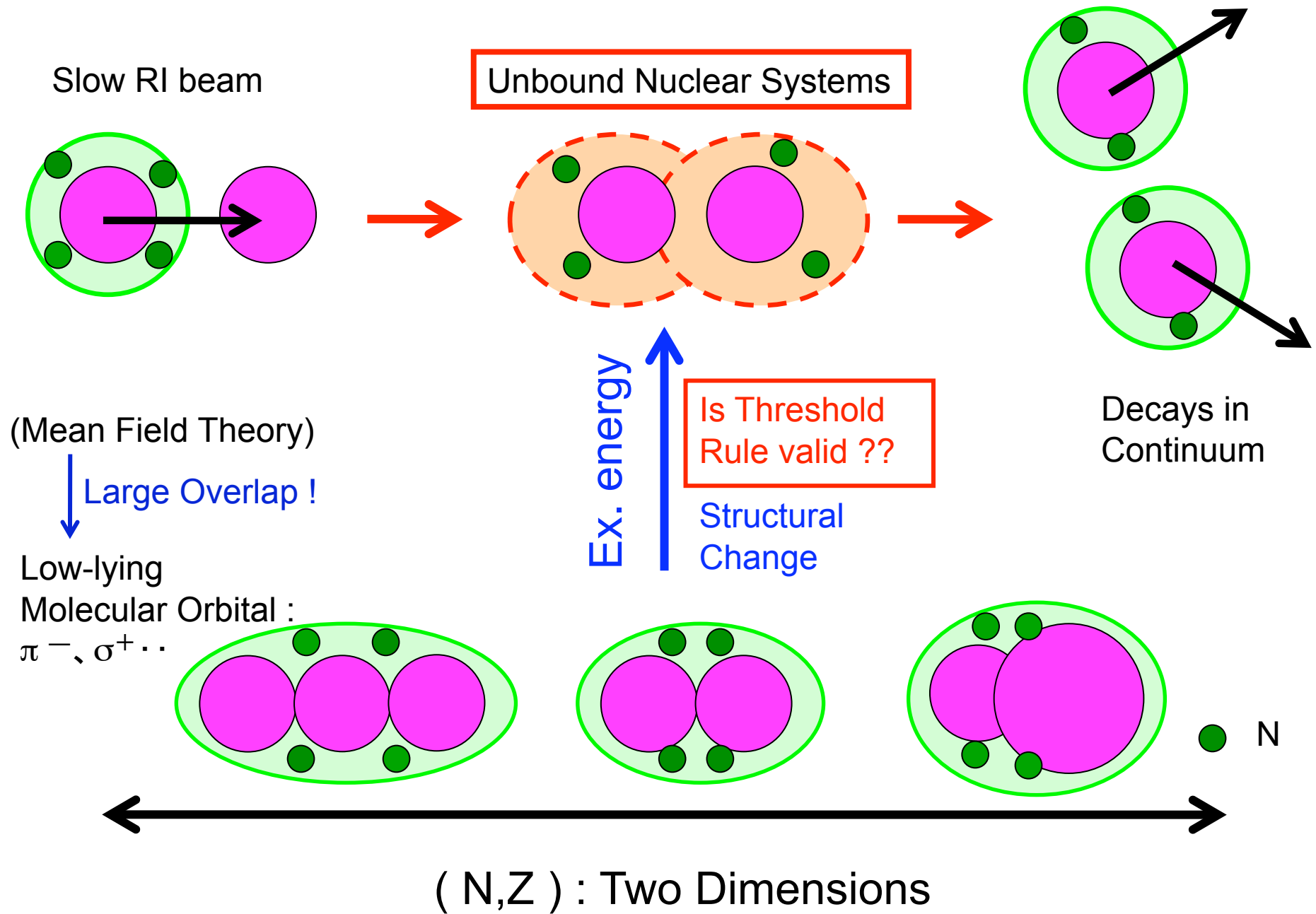
B. Approaches based on the cluster model with effective NN forces

It is important to search for **new cluster-phenomena in unknown systems** by employing **cluster model approaches.**



Neutron Excess Systems

Studies on Exotic Nuclear Systems in (E_x, N, Z, J) Space



^{12}Be (experiments) (Important system before proceeding systematic studies)

Low-lying (Breaking of N=8 Magicity)

High-lying states (Atomic)

$^{11}\text{Be}+n$: 3.17MeV

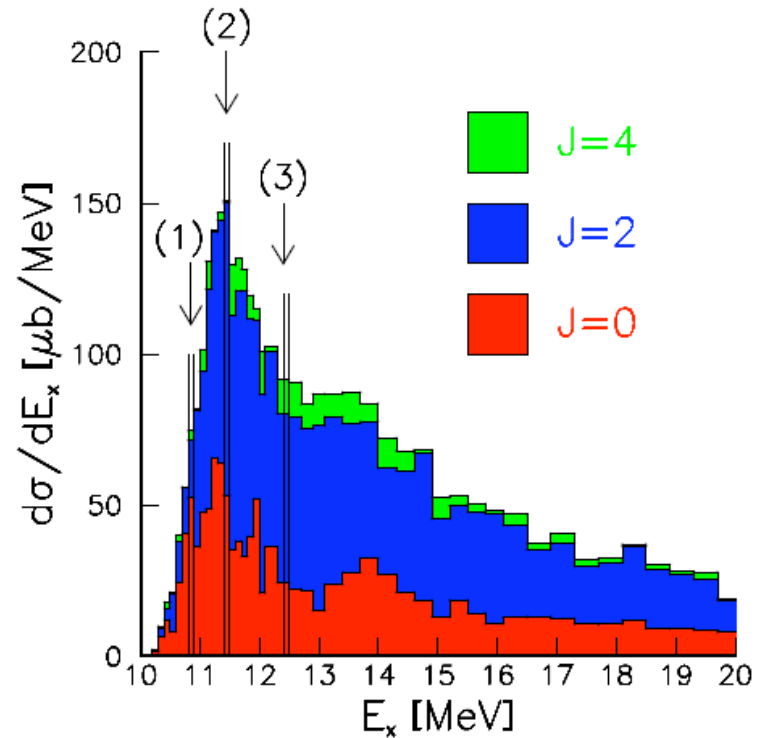
$E(\text{sd-0p})$	<u>2.70</u>	1^-
$\sim 1\text{MeV}$		
	<u>2.24</u>	0^+
	<u>2.10</u>	2^+

Def. Length $\sim 2\text{fm}$

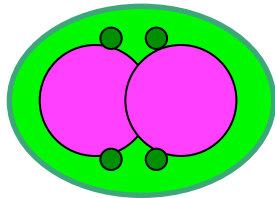
Scattering of
 $^8\text{He} + ^4\text{He} (= ^{12}\text{Be})$
 (Exp. at GANIL)

g.s. 0^+

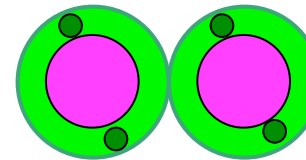
$^{12}\text{Be}+\alpha \rightarrow (^6\text{He}+^6\text{He})+\alpha$: A. Saito et al.



Molecule

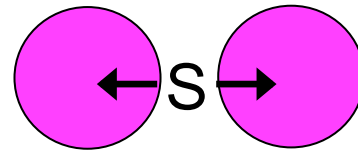


Structural changes

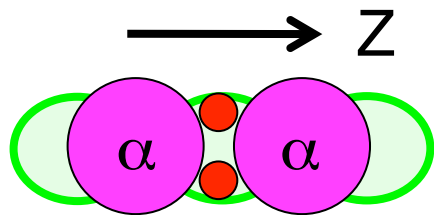


$^6\text{He} + ^6\text{He}$ (Atomic)

Formulation (II)

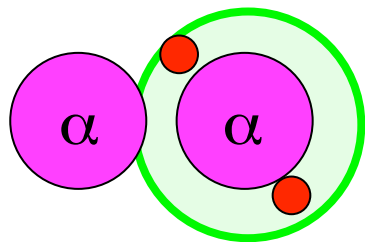


Linear Combination of Atomic Orbital (LCAO)



$$(\sigma^+)^2 = (P_z(L) - P_z(R))^2$$

$$= \underset{\alpha + {}^6\text{He}}{P_z(L) \cdot P_z(L)} + \underset{\alpha + {}^6\text{He}}{P_z(R) \cdot P_z(R)} - \underset{{}^5\text{He} + {}^5\text{He}}{2P_z(L) \cdot P_z(R)}$$



$\alpha + {}^6\text{He}(0^+)$

$$= P_x(R) \cdot P_x(R) + P_y(R) \cdot P_y(R) + P_z(R) \cdot P_z(R)$$

General MO: $(C(L)P_i(L) + C(R)P_j(R))^2$

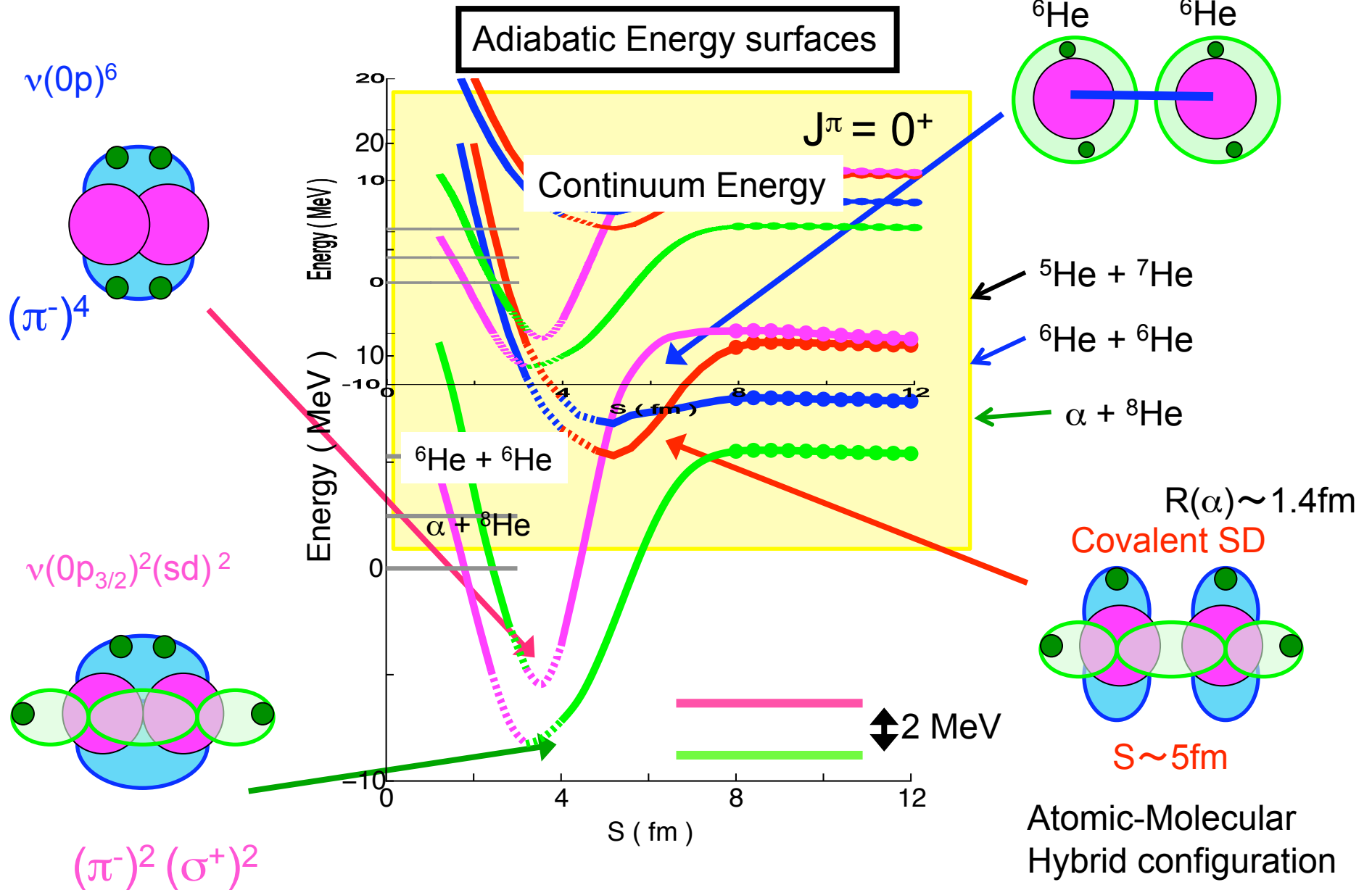
Total wave function

$$\Psi = \sum_{\beta, S} \underline{C(\beta, S)} P_m(\mathbf{a}) \cdot P_n(\mathbf{b}) \quad (m, n) = x, y, z \quad (\mathbf{a}, \mathbf{b}) = L, R$$

Variational PRM

Energy surfaces in $^{12}\text{Be} = \alpha + \alpha + 4\text{N}$ (38 channels)

V_{NN} : Volkov No.2+G3RS



Coupling to open channels in continuum

Closed states method : Prof. Kamimura, Prog. Part. Nucl. Phys. 51 (2003)

Open channels Compound states (Closed)

$$\Psi^{(+)} = \sum_{\beta} \varphi_{\beta} \chi_{\beta,\alpha}^{(+)} + \sum_{\nu} b_{\nu} \Omega_{\nu}^{K=0}$$

Scattering B.C.

↓

$$\chi_{\beta,\alpha}^{(+)} \xrightarrow{R_{\beta}=\infty} u_{L_{\beta}}^{(-)} \delta_{\beta,\alpha} - S_{\beta,\alpha} u_{L_{\beta}}^{(+)}$$

Bound state approximation
with Atomic Orbital Basis

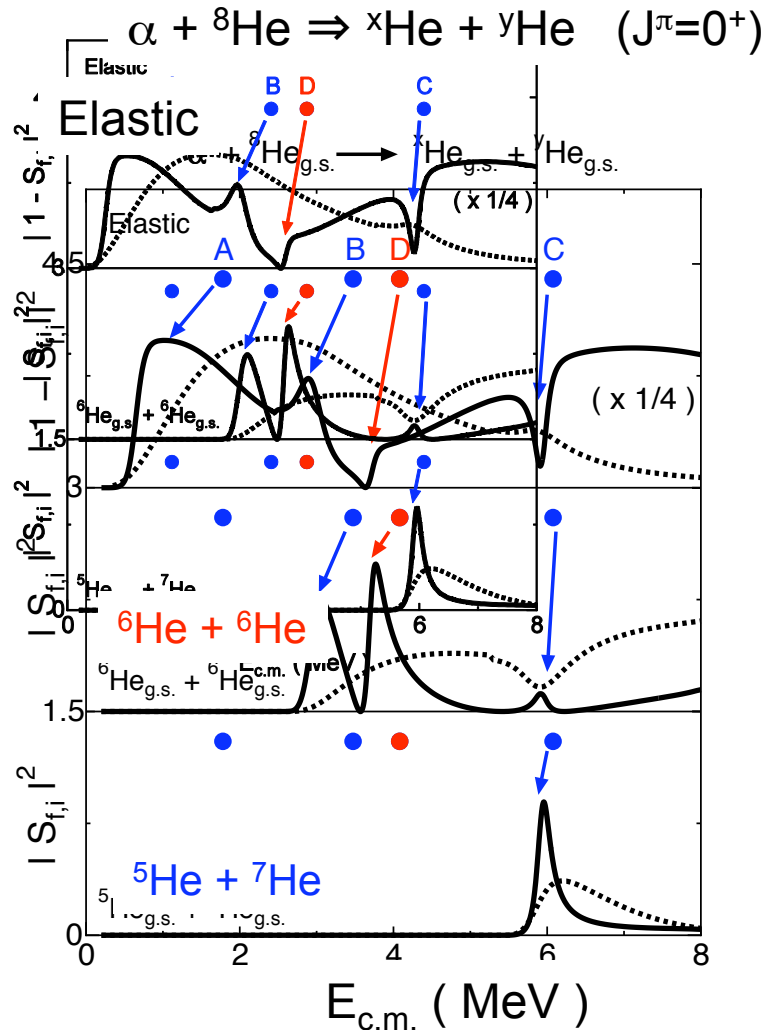
↓

$$(H - E_{\nu}) \Omega_{\nu}^{K=0} = 0$$

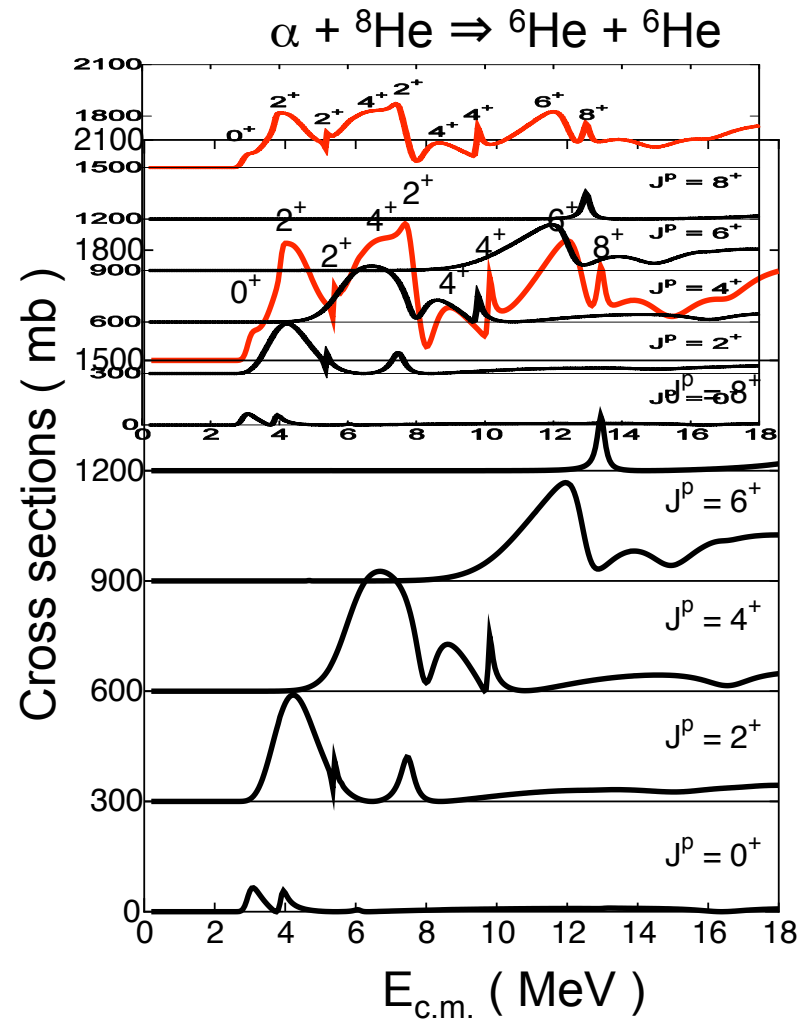
400~500 S.D. with J^{π} projection

Rearrangement channels : $\alpha + {}^8\text{He}_{\text{g.s.}}$, ${}^6\text{He}_{\text{g.s.}}$ + ${}^6\text{He}_{\text{g.s.}}$, ${}^5\text{He}_{\text{g.s.}}$ + ${}^7\text{He}_{\text{g.s.}}$

Cross sections of neutron transfers



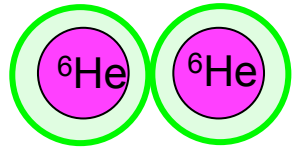
Dotted curves : Three open channels only
 Solid curves: Open + closed channels



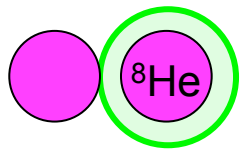
This is a prediction for recent experiments at GANIL.

Excitation modes in ^{12}Be

α - α REL. + S.P. of 4N

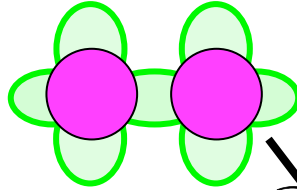


Cluster + S. P. Excitation



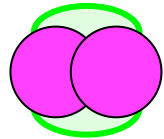
Excitation from the 0_1^+ state.

Covalent SD
($0p_R$)($0p_L$)(σ^+)²

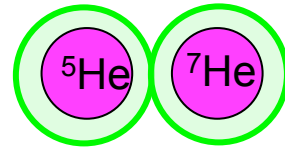
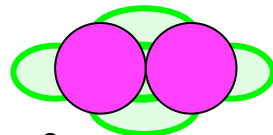


Single particle Excitation

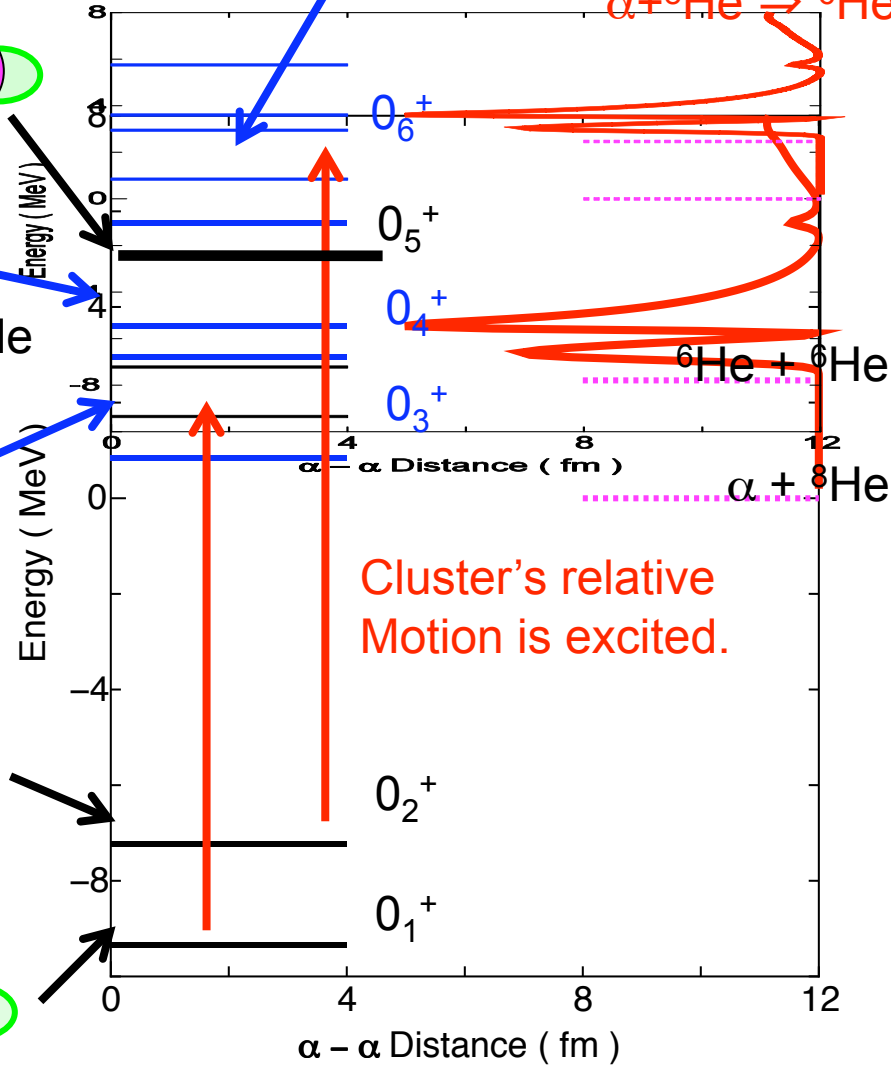
(π^-)²(π^-)²



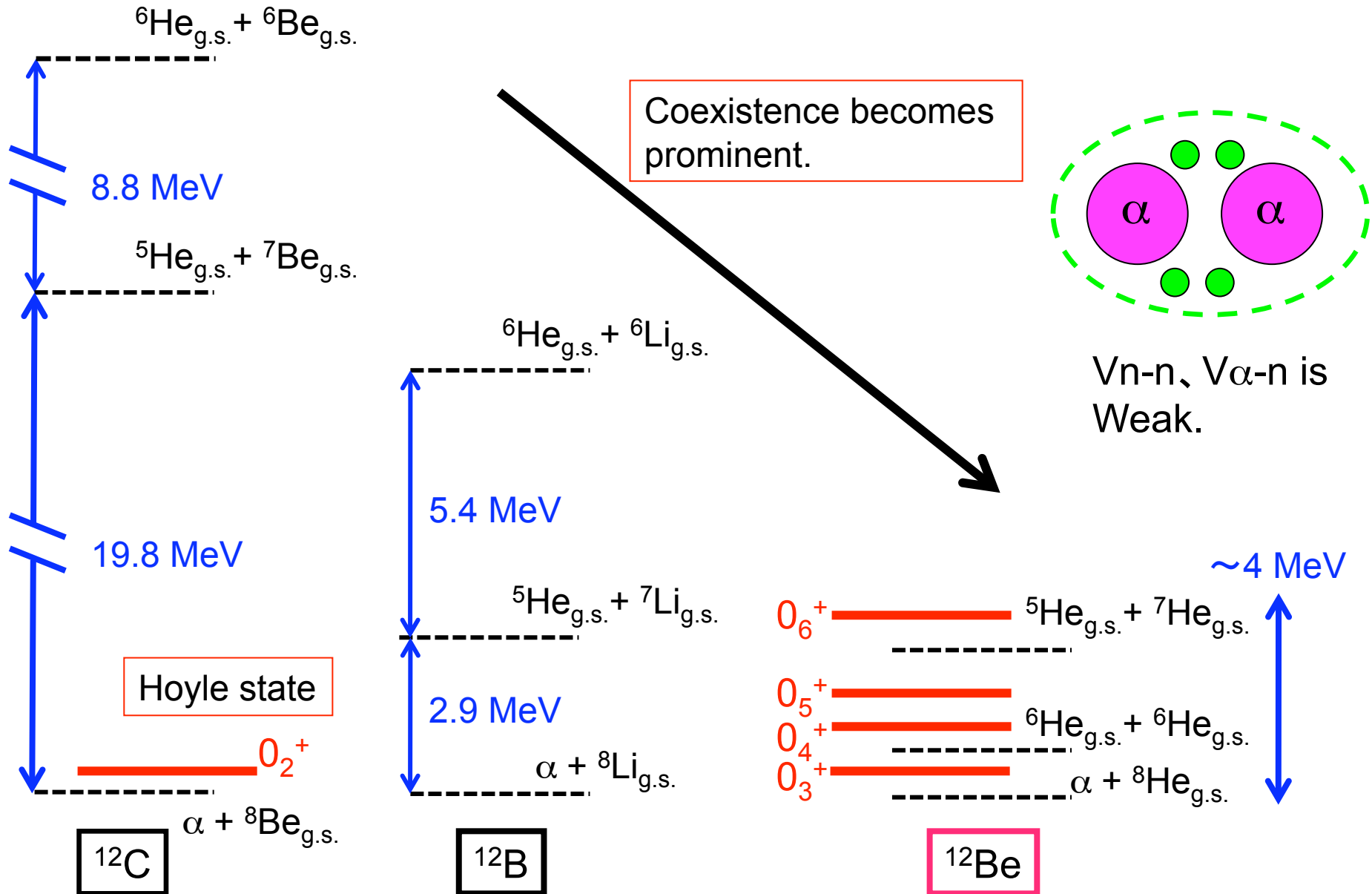
(π^-)²(σ^+)²



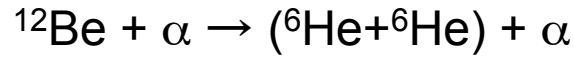
Excitation from the 0_2^+ state.



Coexistence in A=12 systems : Coexistence phenomena

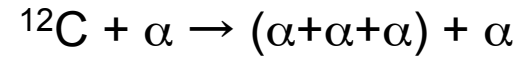
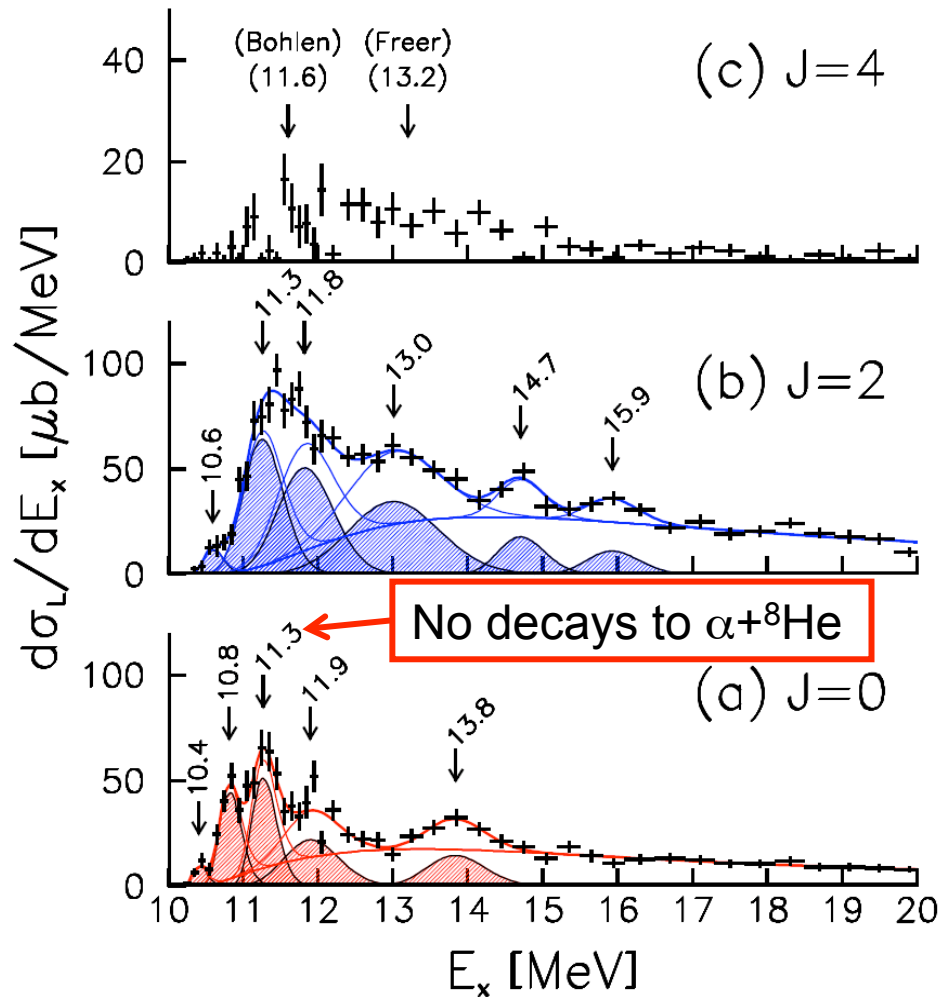


Comparison of ^{12}Be and ^{12}C

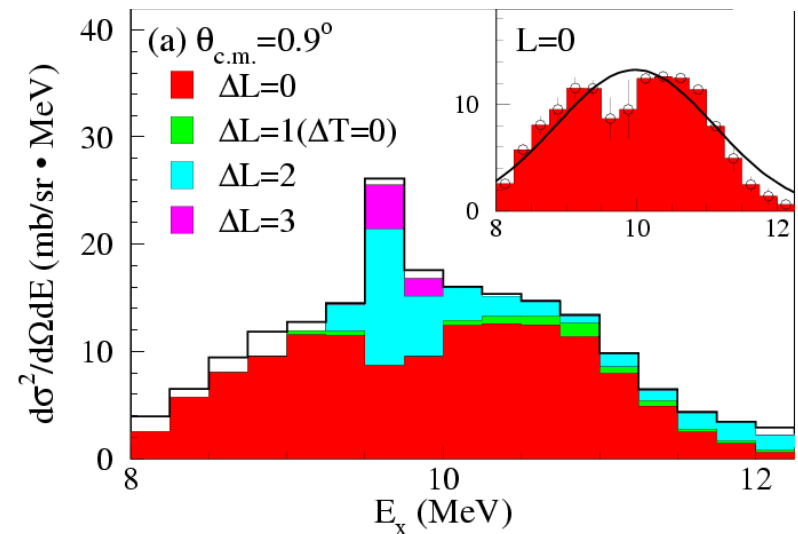
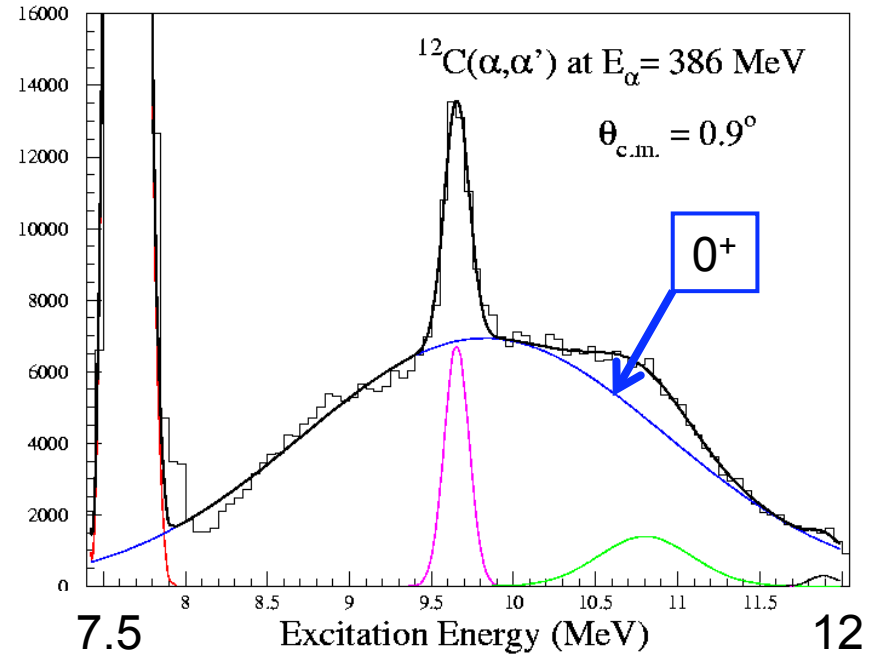


(A. Saito et al. at Tokyo Univ.)

There appear many resonances !!



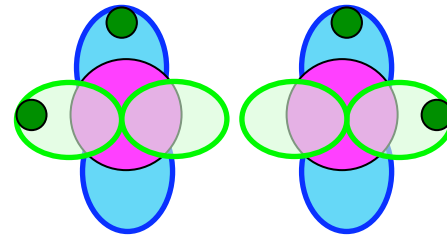
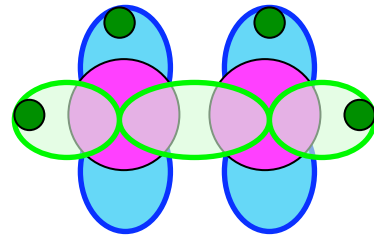
(M. Itoh et al. at Tohoku Univ.)



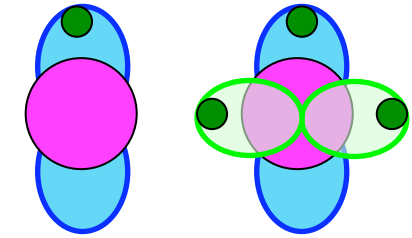
Enhancement of the two neutron transfer

$$|\Phi(Cov.SD)\rangle = |\chi(^6He+^6He)\rangle + |\varphi(^5He+^7He)\rangle$$

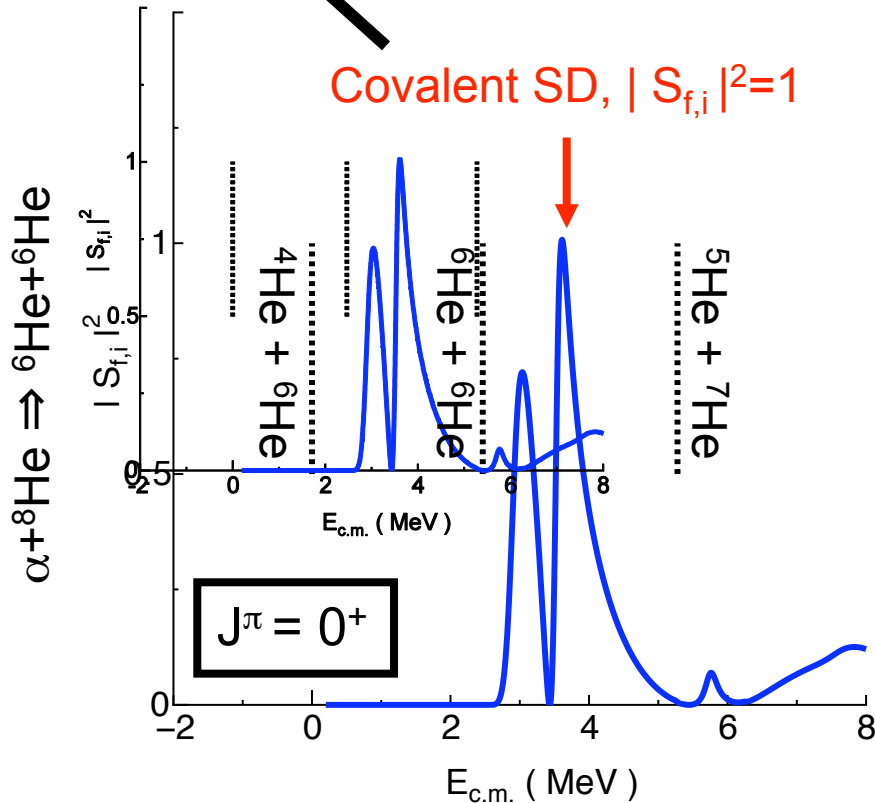
Strong decay into $^6He+^6He$



Open



Closed



Unitary condition of S-matrix

$$\sum_{\beta} |s(\beta \leftarrow \alpha + ^8He)|^2 = 1$$

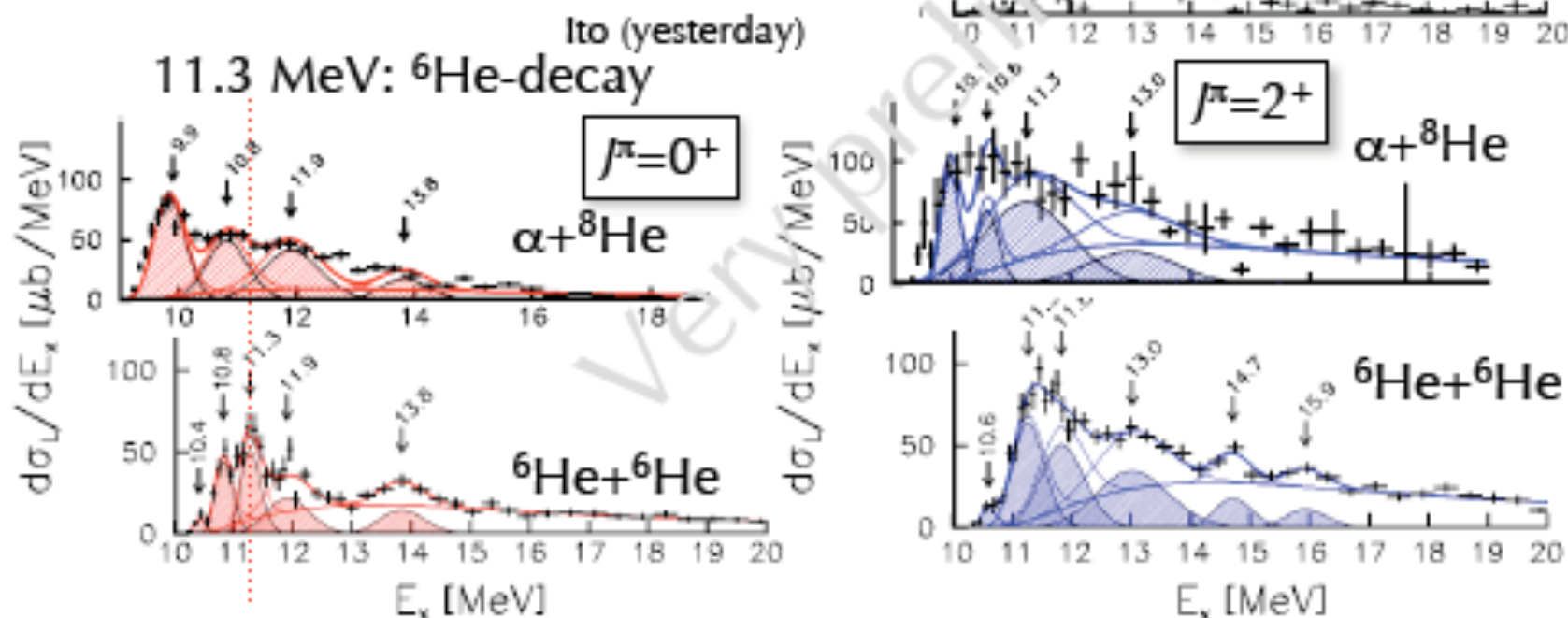
$$|s(^6He+^6He \leftarrow \alpha + ^8He)|^2 \approx 1$$

Large part of the flux flows to $^6He+^6He$.

E_x spectra of $^{12}\text{Be}(\alpha, \alpha')^4\text{He}^8\text{He}$ (positive parities)

- E_x spectra of pos.-parity states
- Fitting with resonances determined in $^6\text{He}+^6\text{He}$ analysis (E_R & Γ_R : fixed for $J=0$ & 2)

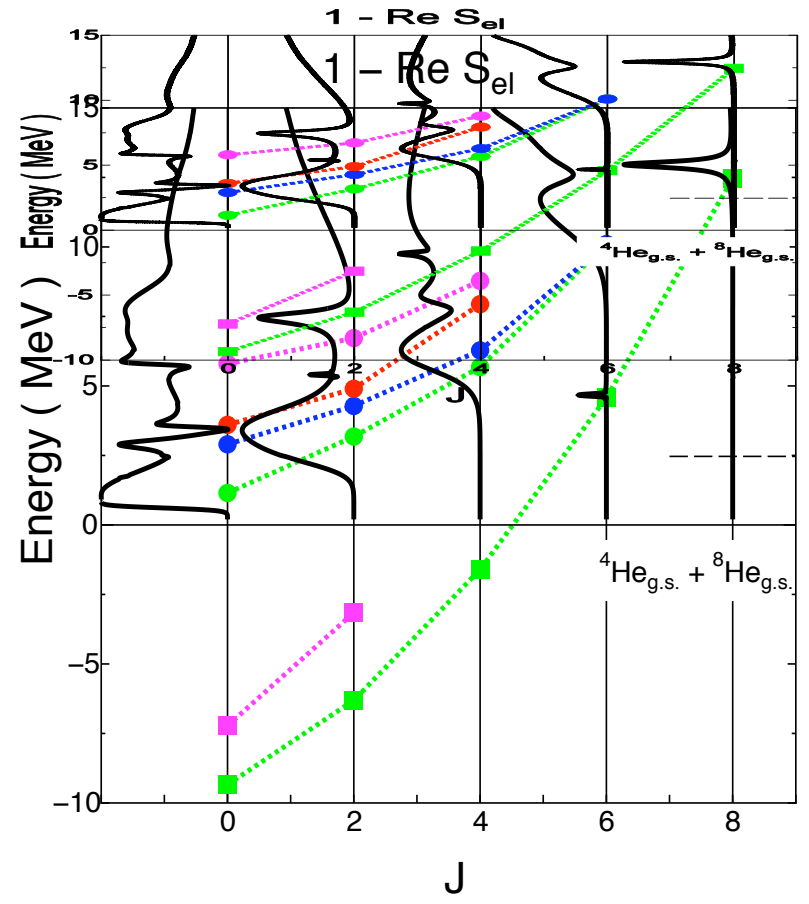
-10 MeV: Korshennikov et al., PLB343('95)53.



Rotational bands : Coexistence of MRs and covalent SD

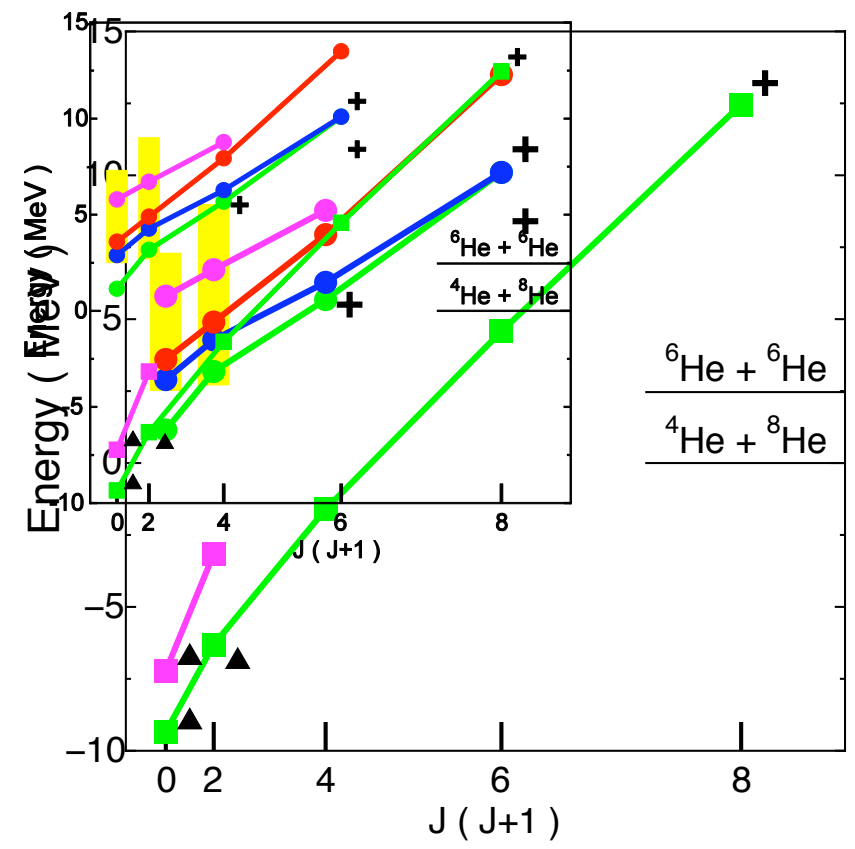
Exp. at RIKEN (Saito)

Exp. by Freer Exp. at RIKEN (Shimoura)



Scattering region

Bound region



Pink : ${}^5\text{He} + {}^7\text{He}$

Green : ${}^6\text{He} + {}^6\text{He}$

Red : Covalent SD

Blue : $\alpha + {}^8\text{He}$

Green squares: $(\pi^-)^2(\sigma^+)^2$

White square: $(\pi^-)^2(\pi^-)^2$

Monopole Transition

Why monopole ?

There is a possibility that monopole transitions are enhanced
If cluster structures are developed. ($E_x < 10\text{MeV}$)

Cluster structure



Pioneering work on monopole transition

$$M(E0, IS) = \left\langle 0_1^+ \left| \sum_{i=1}^A r_i^2 \right| 0_{ex}^+ \right\rangle$$

Cluster correlation in a ground state (Yamada et al., PTP, inpress)



Excitation of clusters' relative motion ($2\hbar\omega$)

$$M \approx \left\langle G.S. \right| A_{rel}^+(2\hbar\omega) \left| Cluster \right\rangle$$

If $\left\langle G.S. \right|$ has large cluster components, the monopole matrix elements
will be enhanced !

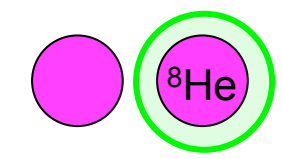
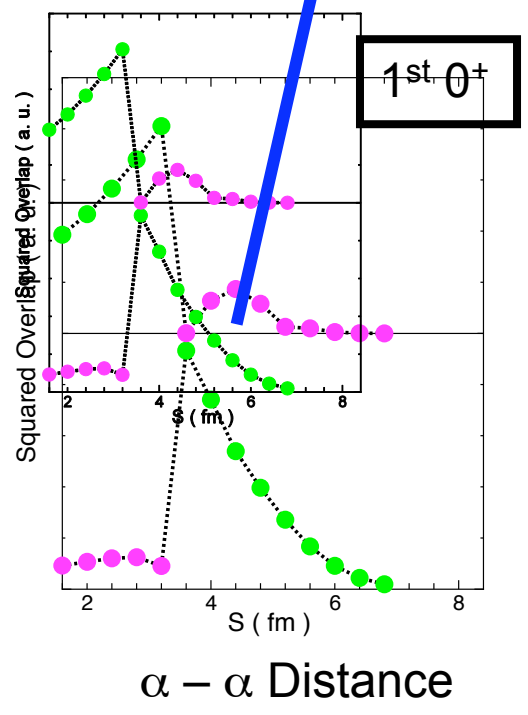
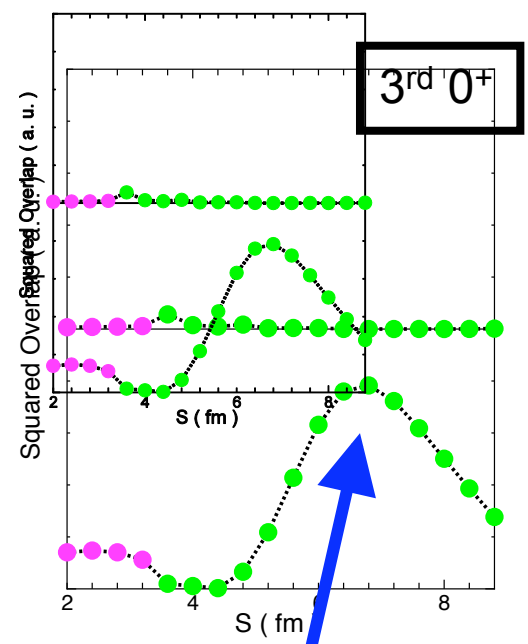
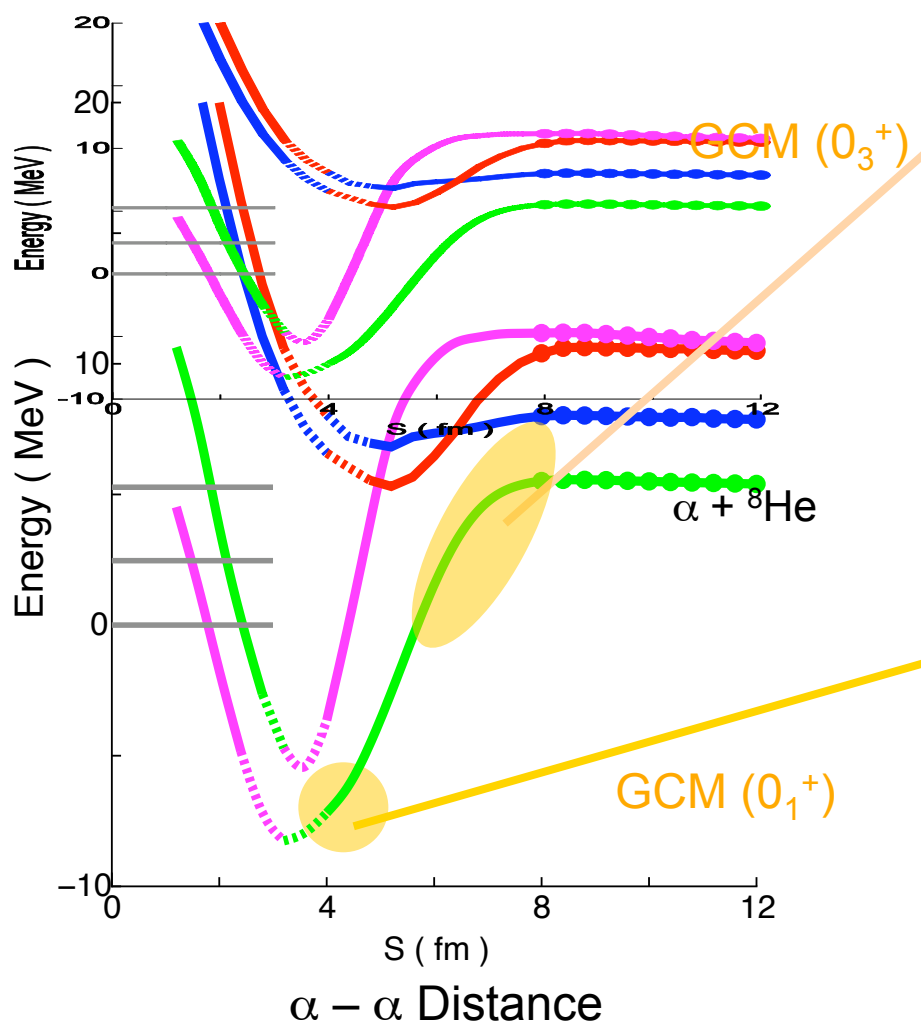
Large cluster components in G.S. can be always justified by Bayman-Bohr Theorem

Simple shell model (1p-1h, 2hw) : No strength around low-lying region, $E_x < 10\text{MeV}$

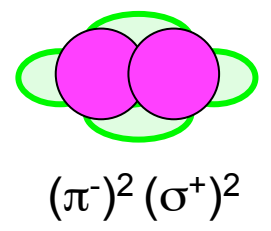
Adiabatic energy surfaces in ^{12}Be

V_{NN} : Volkov No.2+G3RS

Adiabatic Energy surfaces



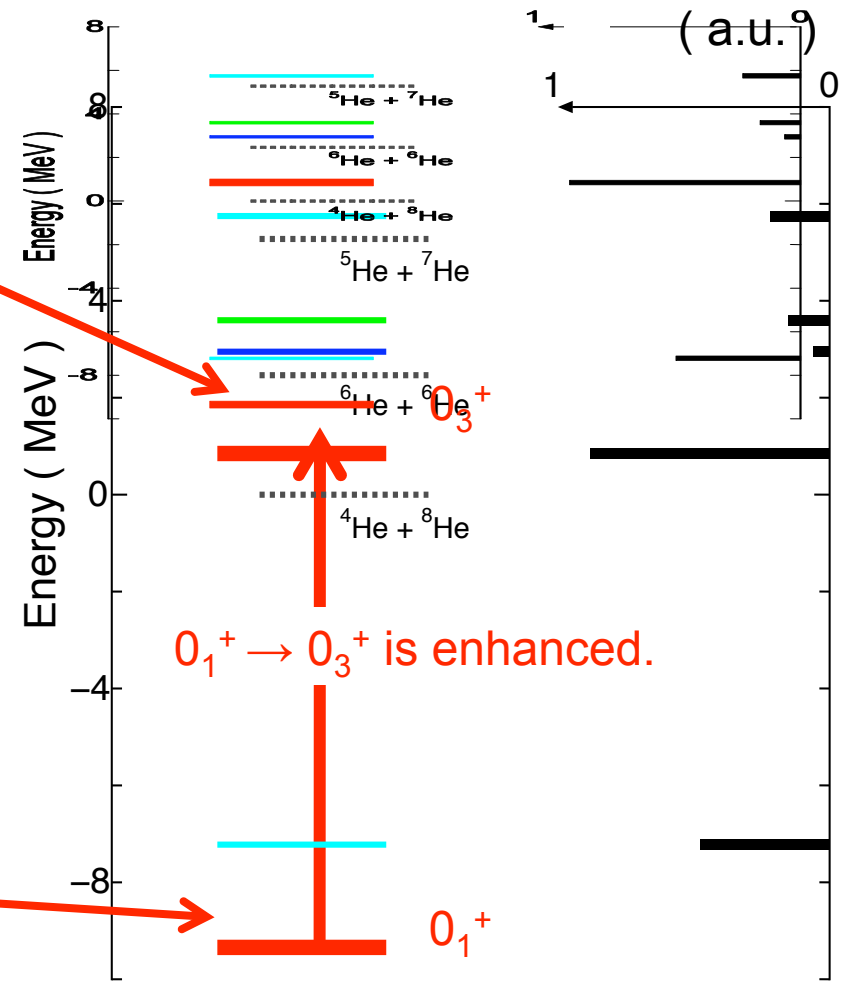
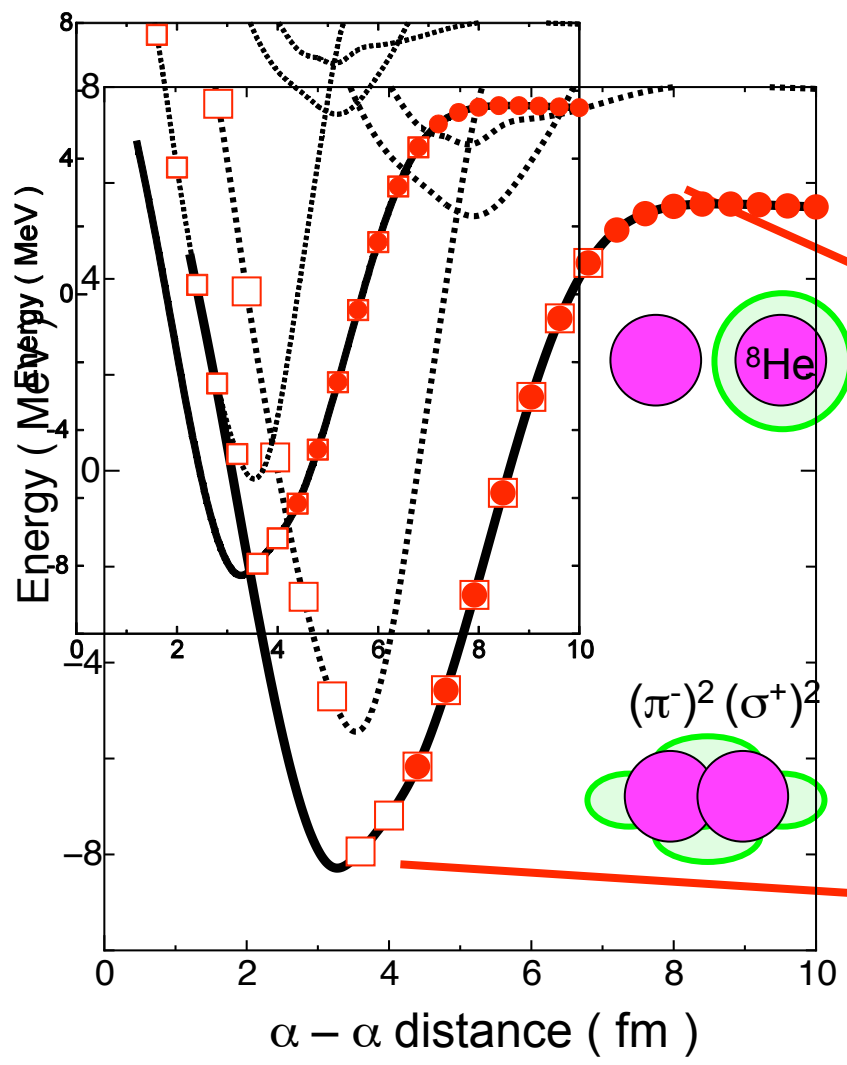
Cluster Excitation



Monopole transition of ^{12}Be

$$M(E0, IS) = \left\langle 0_f^+ \left| \sum_{i=1}^A r_i^2 \right| 0_1^+ \right\rangle$$

Adiabatic connection enhances the Monopole transition !

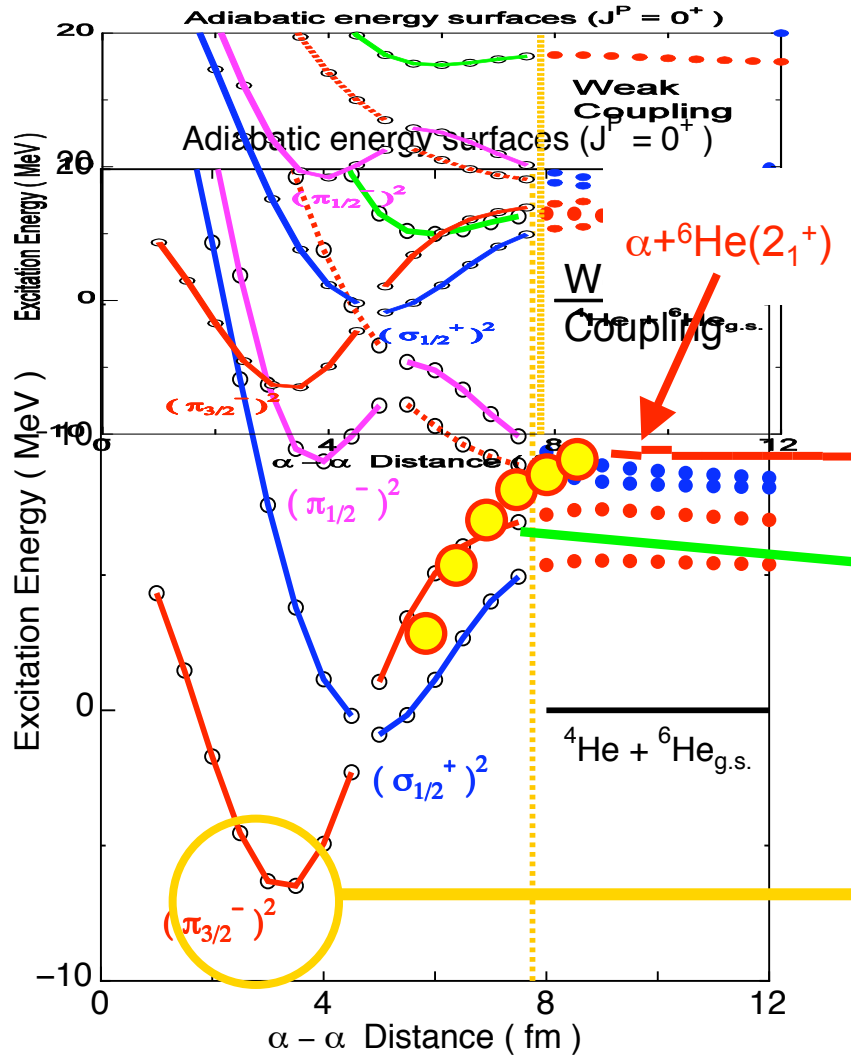


$0_1^+ \rightarrow 0_3^+$ is enhanced.

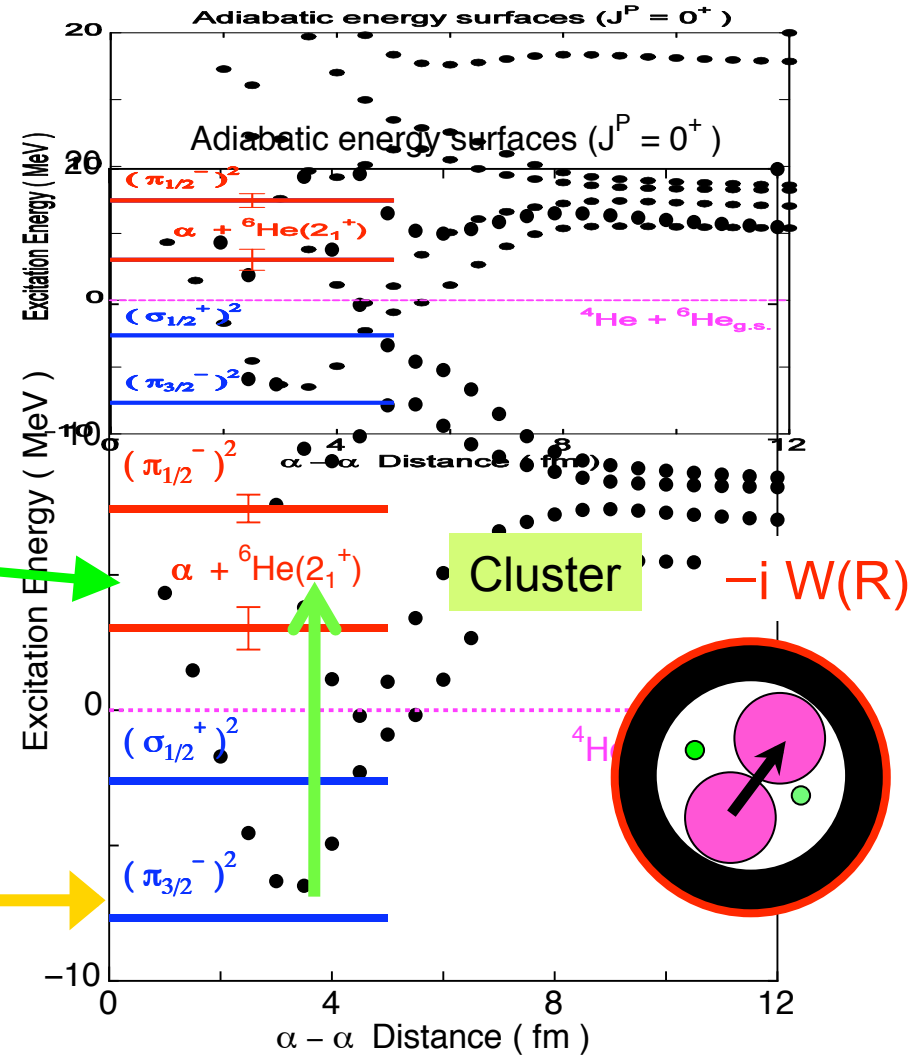
$$|M(E0, IS)|^2$$

(a.u.)

Adiabatic surfaces ($J^\pi = 0^+$)



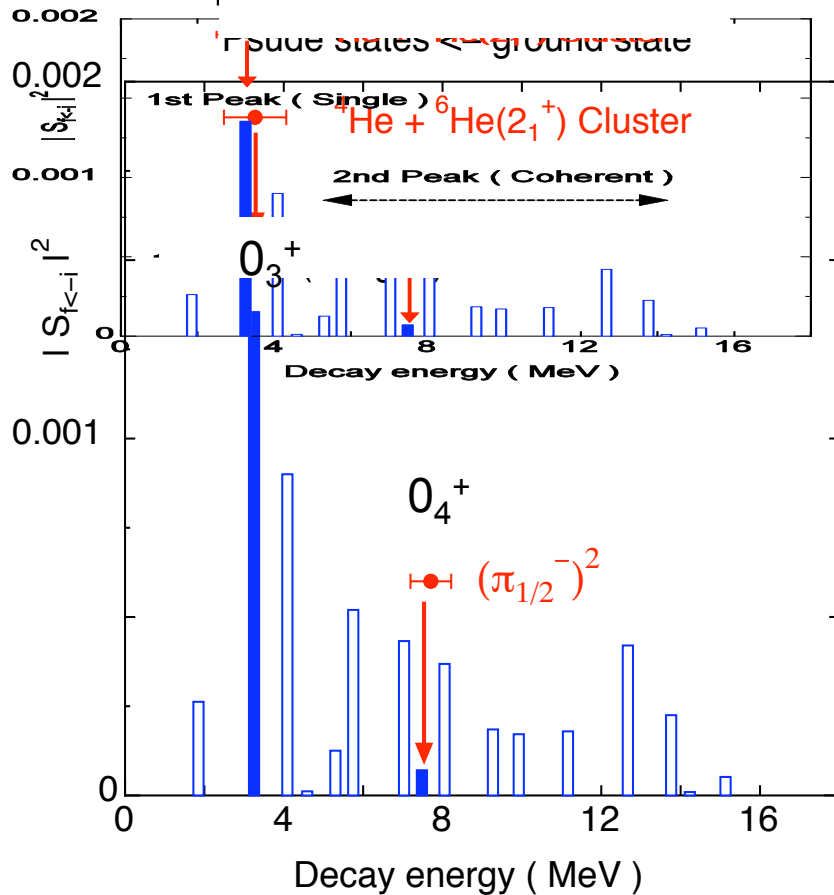
Energy spectra ($J^\pi = 0^+$)



Nuclea breakup : $^{10}\text{Be} + ^{12}\text{C} \Rightarrow ^{10}\text{Be}(0^+ \text{ conti.}) + ^{12}\text{C}$ (CDCC)

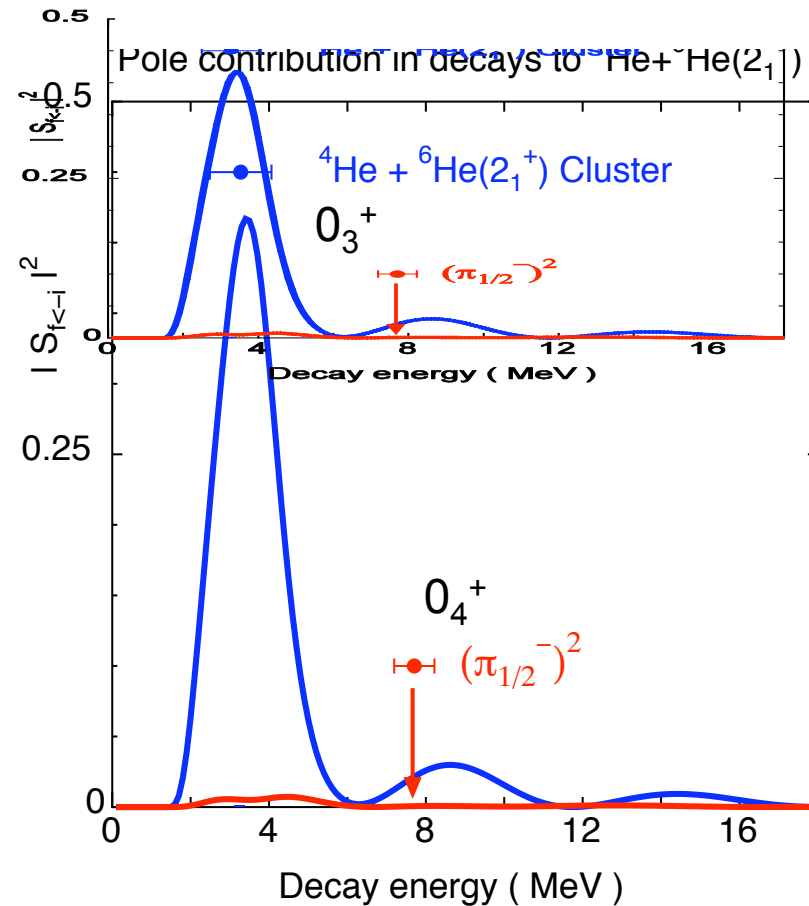
Smatrix (Conti. ← G.S.)

S-matrices to continuums



Smatrix (Poles ← G.S.)

S-matrices to Poles



New aspects in the calculation

PRL100 (08), PRC78(R) (08), Phys. Rev. Focus 22(08)

Large scale coupled channels in the (two-body) microscopic cluster model

Present calculation: 38ch \Leftrightarrow Previous one: a few channels

New pictures obtained from the calculation (Coexistence)

1. Picture of ground state

\Rightarrow It should be considered as the hybrid state, in which **the clusters and single particle state (of valence neutrons) coexist**. (Different from the simple M.F. state)

2. Level scheme in the unbound states

\Rightarrow **Unbound states coexist with a small energy interval** due to the weak interacting Properties of neutron-excess systems. (Phenomena other than halos or skins.)

Feature studies

These properties will appear **systematically in other light neutron excess system**.

Systematic study \Rightarrow $O = \alpha + {}^{12}\text{C} + \text{XN}$, $\text{Ne} = \alpha + {}^{16}\text{O} + \text{XN}$

Measurement of the degenerating levels \Rightarrow High resolution system (SAMURAI ?)

Collaborators and encouragements

Hokkaido University : Profs. Kiyoshi Kato, Akira Ohnishi

Kyushu University : Prof. Masayasu Kamimura

RIKEN : Profs. Kiyomi Ikeda, Takashi Nakatsukasa, Hiroyoshi Sakurai

Tokyo University : Prof. Naoyuki Itagaki

Tsukuba University : Prof. Kazuhiro Yabana

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[Phys. Rev. C 78, 011602](#)

(issue of July 2008)

[Phys. Rev. Lett. 100, 182502](#)

(issue of 9 May 2008)

[Titles and Authors](#)

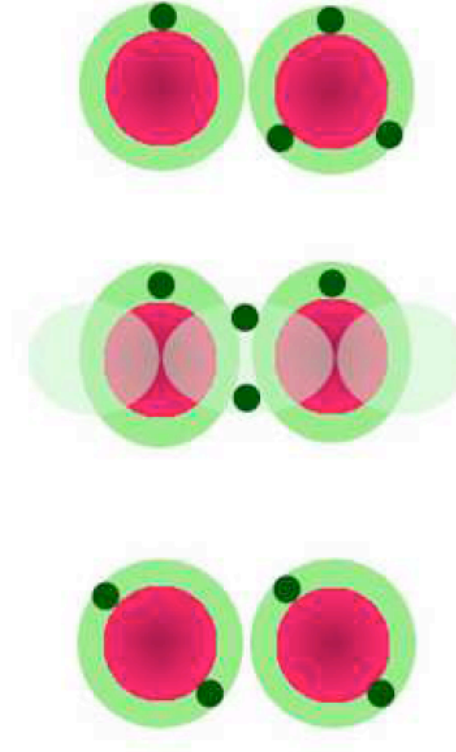
25 July 2008

(<http://focus.aps.org/story/v22/st4>)

Nuclear Chemistry

An atomic nucleus can sometimes look like a kind of molecule--a pair of particle clumps bound together like a pair of atoms, except that neutrons take the place of electrons as the "glue." In the 9 May *Physical Review Letters* and in the Rapid Communications section of the July *Physical Review C*, Japanese theorists show that a single beryllium nucleus can briefly resemble a covalently bonded molecule, an ionically bonded molecule, or just a pair of neutral atoms. It all depends on the energy with which two smaller nuclei collide to create the beryllium nucleus. The results underscore the importance of clumps of neutrons and protons within nuclei.

The alpha particle, containing two protons and two neutrons, plays a special role in the structure of nuclei. This tightly bound quartet emerges during radioactive decay of nuclei like uranium-238, and is itself the nucleus of the common isotope of helium. When two helium nuclei containing extra neutrons collide, the alpha particles may



adapted from M. Ito/RIKEN

See also,
RIKEN RESEARCH
5 September (2008)

<http://www.rikenresearch.riken.jp/research/517/>